Wake Fields and Beam Dynamics

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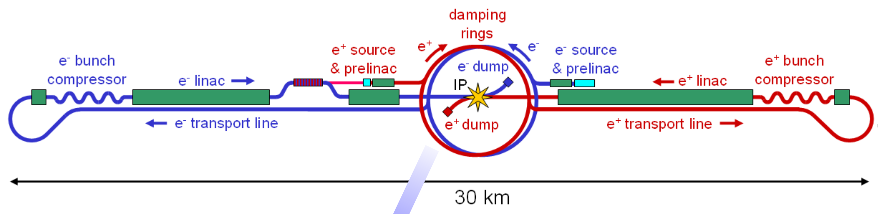
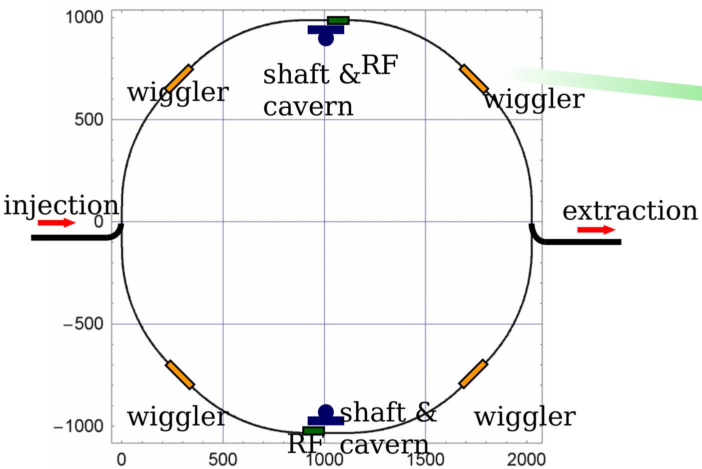
Overview

- Research Interests
  - Wake fields
    - Electromagnetic fields are induced by charged particles interacting with beam pipes, rf cavities, etc.
  - Multi-bunch instabilities
    - Wake fields perturb trailing particles, causing them to deviate from the reference trajectory
  - Transient effects
    - Injection of particles generates strong wake fields that induce jitter in damping rings

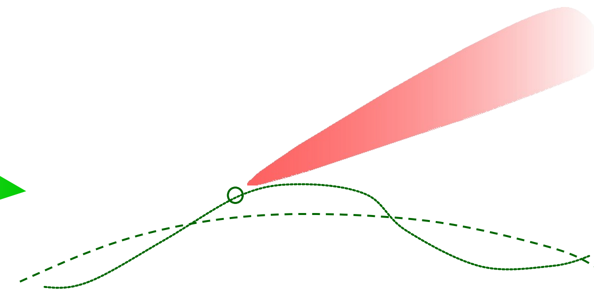
Damping Ring



An accelerator



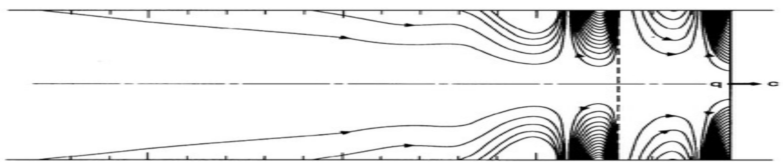
A damping ring



- Synchrotron radiation dampens beam oscillation
- Produces extremely stable, narrow beams ~ μm

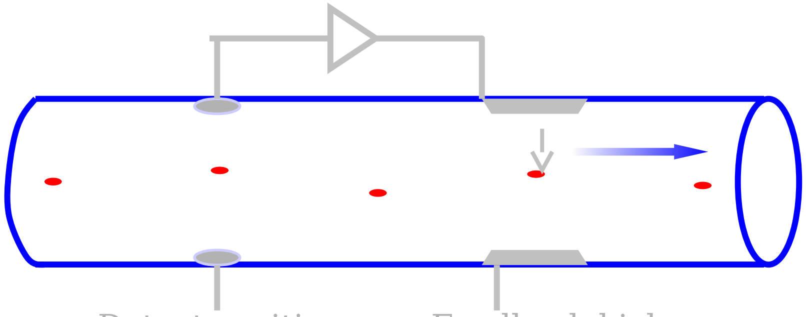
Multi-bunch instabilities

ke field is the E field trailing behind a charged particle.



(Karl Bane 1991)

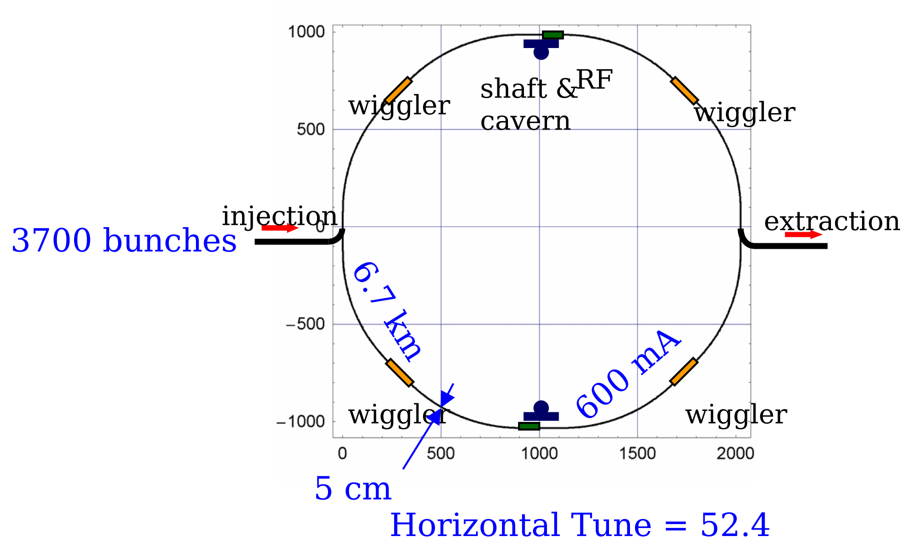
icles behind perturbed – oscillations grow, particles hit the



ack system needed to generate sufficient kick to control osc

The problem of coupled bunches

For a damping ring with these parameters:



Wake field causes bunch amplitudes to grow at a rate of 1.4% per turn.

For a beam size of 1 um,
the beam will hit the wall after 710 turns,

or 1 second.

Feedback System is Required

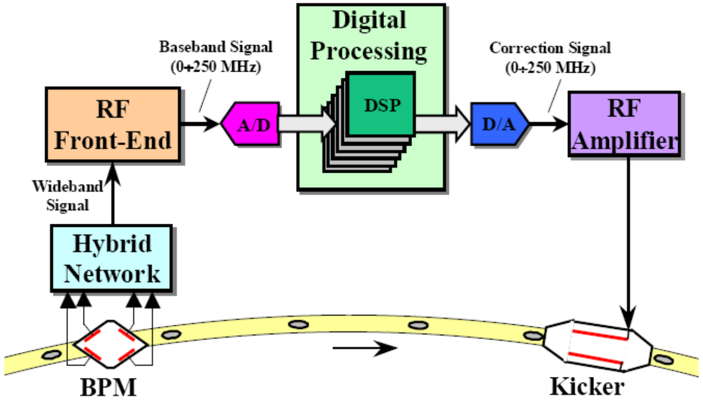


Figure 1: Block diagram of the Transverse Multi-Bunch Feedback System

(Lonza, Bulfone, Gamba 1999)

- A simple schematic of a feedback system
  - Beam position monitor (BPM) detects position of bunch, electronic system processes signal, and kicker …
  - Kicker provides a deflection to bring the bunch back towards the desired trajectory

Time domain simulation

Feedback system limited by technology to provide sufficiently fast response to individual bunches

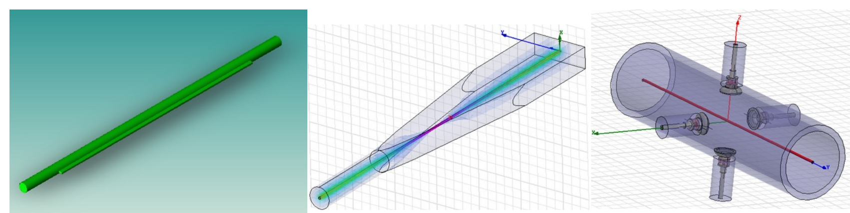
Accurate prediction of growth rates of unstable oscillation needed. Standard analytic formula exists, but assumes constant focusing field around the ring

Real lattice contains many separate magnets, produces
complex beam dynamics – requires time domain simulation

Essential ingredients: [7]Wake functions

Wake Fields

- In a real accelerator
  - Factors
    - wake fields are determined by the geometry and materials of the environment around the beam.

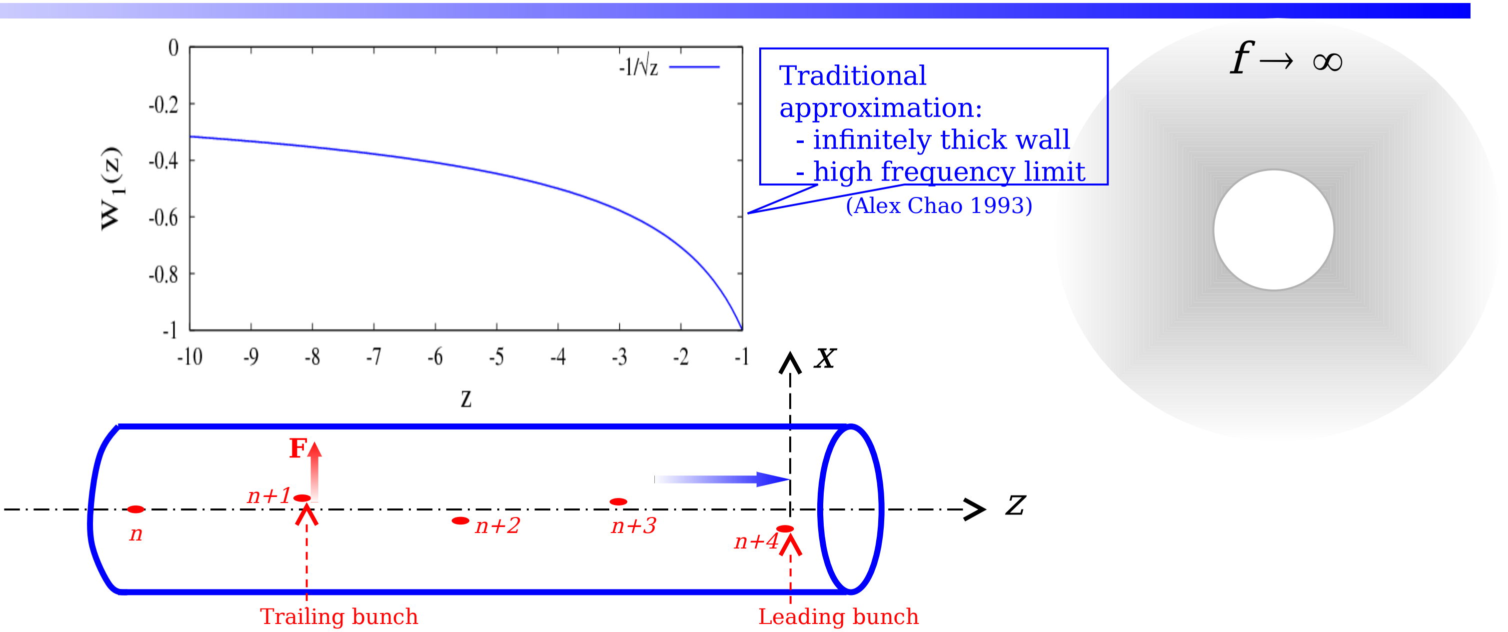


Figures by Maxim Korostelev 2008

  - Range
    - short-range effects: within a single bunch
    - long-range effects: between different bunches

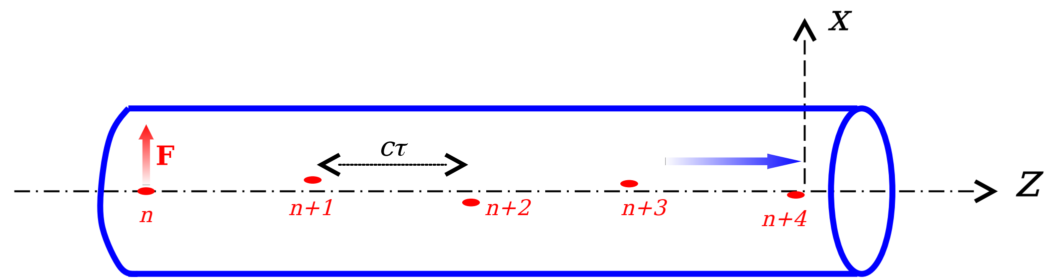
Iain Focus – uniform beam pipe, long range effects.

Uniform Beam Pipe



- Mathematical tools to calculate wake fields:
  - Wake function
    - gives the force on a bunch as a function of distance from a leading bunch that is offset from the desired path by unit displacement.
  - Impedance
    - dynamical analysis starts in the frequency domain, where the Fourier transform of the wake function is known as the impedance.

Equations of motion

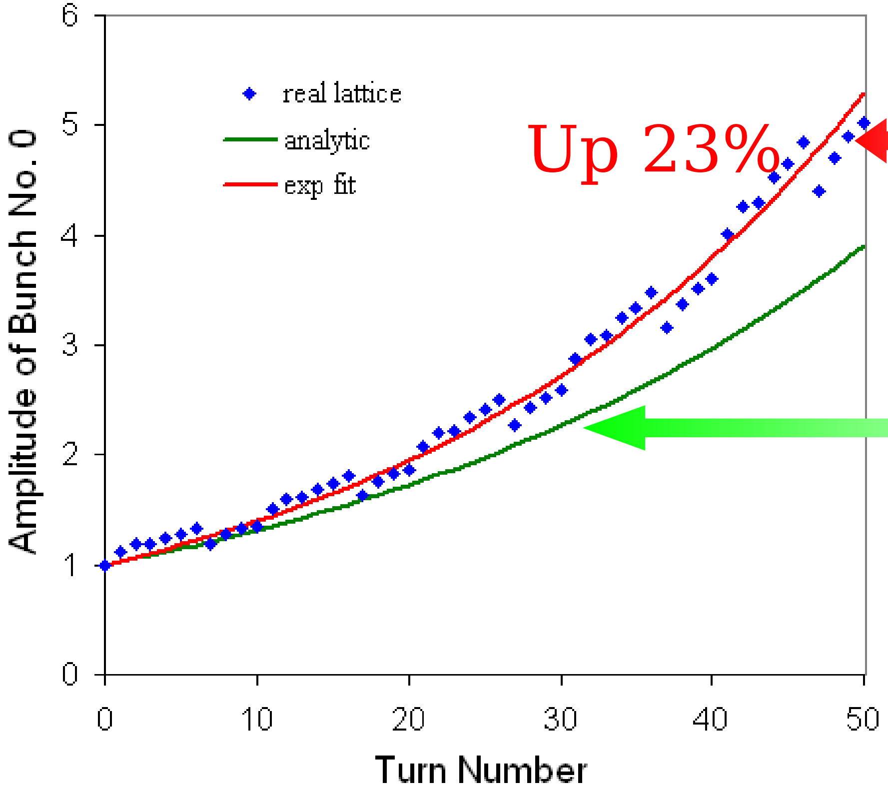


$$\ddot{x}_n(t) + \underset{\substack{\uparrow \\ \text{Magnet strength}}}{K(t)}\,x_n(t) = -\frac{N r_0 c}{\gamma T_0}\underbrace{\left[W_1(-c\tau)x_{n+1}(t-\tau) + W_1(-2c\tau)x_{n+2}(t-2\tau) + W_1(-3c\tau)x_{n+3}(t-3\tau) + \dots\right]}_{\text{Wake sum}}$$

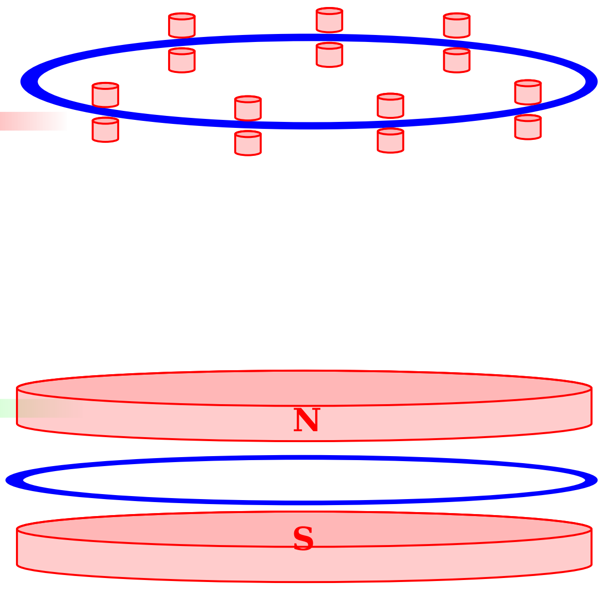
- Focus on transverse motion
  - Assume no coupling between transverse (x) and longitudinal (z) motion.
  - In real accelerator, though only approximately true,
    this model is supported by experimental evidence.

Simulation Results

Determining growth rates



Actual lattice



Traditional approximation

- Can provide more accurate specs for feedback system design
- For further improvement: correct wake function needed

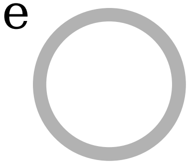
Hock and Wolski, Phys. Rev. ST AB 10, 084401 (2007)

The wall has finite thickness

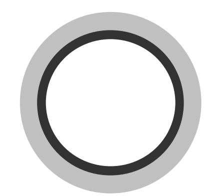
- Traditional approximation for resistive wall wake field assumes wall of pipe infinitely thick

- Traditional formula for strength of wake field (wake function) valid only at high frequency $\sim \frac{1}{\sqrt{z}}$

- Need for accurate growth rate motivates the use of precise wake function when wall is finite ~ 2 mm



- NEG coating used to improve vacuum has high impedance – could increase wake function, requires multilayer calculation



- Published algorithms exist but incomplete. Have to start from first
  principles and solve Maxwell's equations.

Solving Maxwell's equations

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

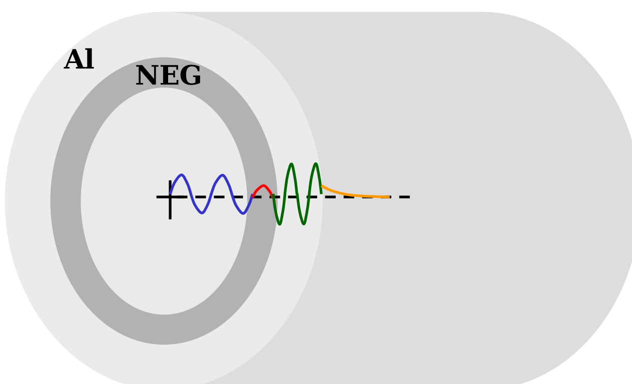
$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

Wave equations
Bessels functions
boundaries
etc

$$\begin{aligned}
a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1N}x_N &= b_1 \\
a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2N}x_N &= b_2 \\
a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \cdots + a_{3N}x_N &= b_3 \\
\cdots \quad & \quad \cdots \\
a_{M1}x_1 + a_{M2}x_2 + a_{M3}x_3 + \cdots + a_{MN}x_N &= b_M
\end{aligned}$$

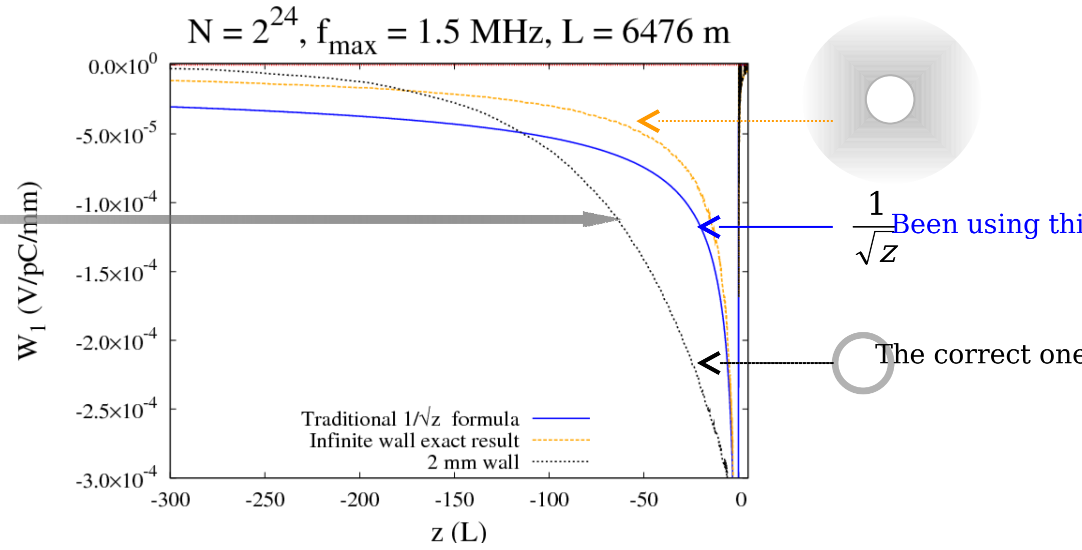


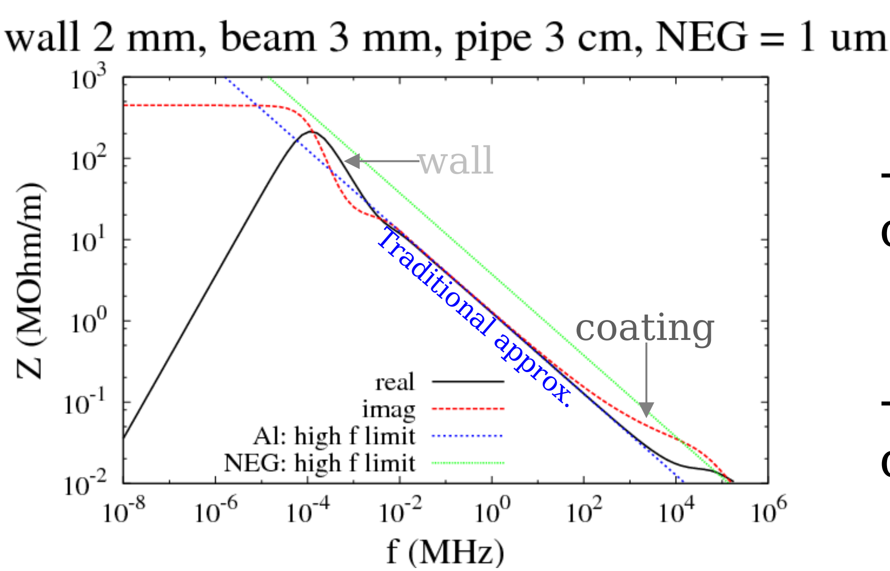
- Field decays rapidly through metal wall
- Huge differences in a_{ij} give badly conditioned matrix
- Arbitrary precision arithmetic required for computation

Wake function results

2 mm wall wake function much larger at distances required for wake sum to converge

→ growth rates much larger than 23% !





- Frequency domain shows distinct effects of wall and coating

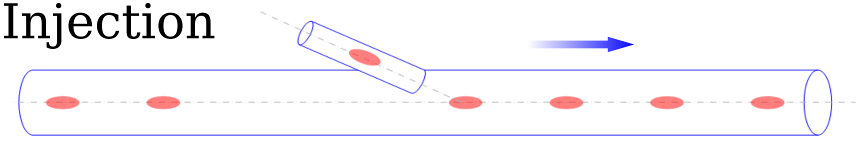
- At low frequency, very different from traditional approximation

Next steps

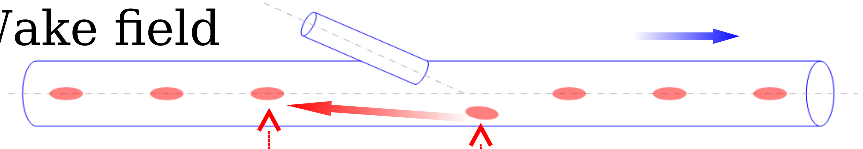
- Resistive wall wake field
  - analytic formulae: low frequency limit, thin coatings
  - Numerical precision check. Simpler algorithms, formulae
- Multi-bunch instabilities
  - Improve reliability, convergence tests (wake, lattice function)
  - Coupling and nonlinear effects?
- Transient effects of beam injection
  - Minimise jitter: correlation with fill patterns?
  - Further optimisation of time domain simulation

Transient effects

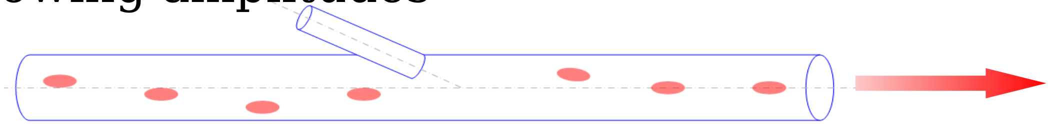
Injection

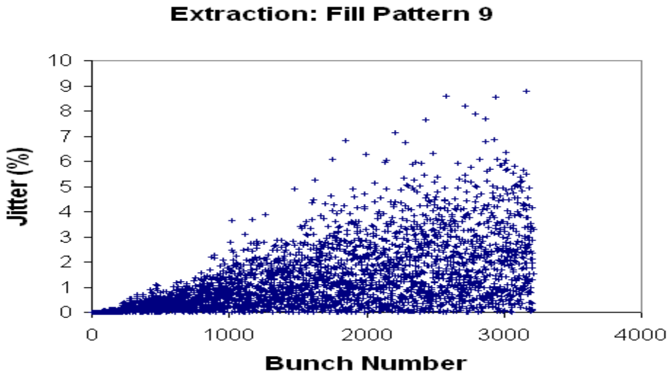


Wake field



Growing amplitudes





- Problem, if we want low emittance (μm beam)
- Tools developed now capable of simulating this
- Explore ways to minimise jitter

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Future work

- Generic Research in Accelerator Physics
  - Explore new physics regimes at higher beam energies, lower emittance, …
  - Develop new experimental tests for predictions of simulation results

Measurements at KEKB

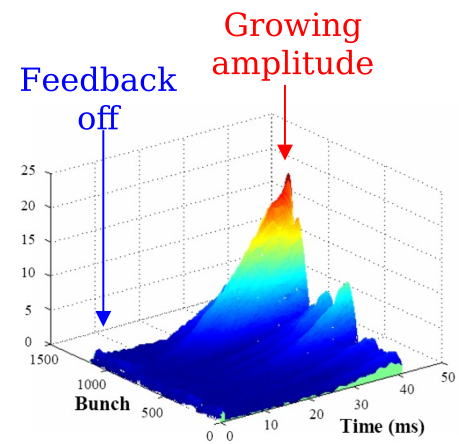

Fig. 3 The horizontal amplitude growth of bunch oscillation in HER at 400 mA

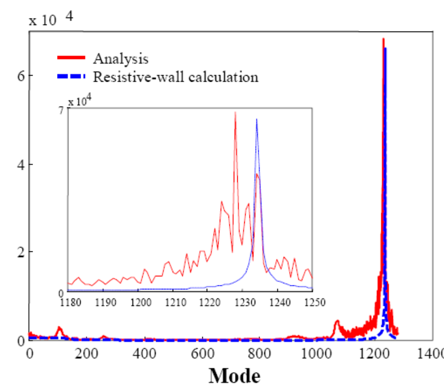

Fig. 1 The horizontal mode distribution of HER at 700 mA
(Win, Fukuma, Kikutani, Tobiyama 2001)

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